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On the prediction of residual strength for notched laminates

GIANCARLO CAPRINO
Istituto di Tecnologie, Università di Napoli, Naples, Italy

The effectiveness of a two-parameter model, based on linear elastic fracture mechanics concepts, is experimentally verified, using original results and data available from the literature. It is shown that the proposed model can be reduced to a one-parameter model, due to the constancy of the other parameter for fibre dominated laminates, irrespective of fibre and matrix materials and shape of the notch. A comparison with the well-known criteria proposed by Whitney and Nuismer (WN) shows that the WN models are useful for prediction of residual strength of notched laminates, provided the length of the notch is not too large; for large notches the two-parameter model reported here seems to be more effective.

1. Introduction

Because of their intrinsic brittleness, composite materials have been extensively studied with regard to their notched strength.

At present two different approaches are available to predict the residual strength of notched laminates: the Waddoups, Eisenmann and Kaminski (WEK) model [1] and the point and average stress criteria proposed by Whitney and Nuismer (WN) [2, 3].

The possibility of using these approaches for design calculations is linked to the measurement of a characteristic dimension (a^* in the WEK model, d_0 in the point stress criterion, and a_0 in the average stress criterion) assumed to be a material property independent of laminate construction and notch or hole length.

The WEK and WN criteria are conceptually very similar: it has been shown [4] that the WEK model is identical to the average stress criterion, since it turns out that $a^* = a_0/2$.

Recently, some researchers have reported [5, 6] that the characteristic lengths are not independent of notch length; and as a consequence the models mentioned above would not be able to describe the notch length effect.

This work presents a very simple two-parameter model, based on linear elastic fracture mechanics concepts. It is practically coincident with the formula proposed by Mar and Lin [7] but it also provides a physical explanation for the quantities there involved.

Using original results and data available from the literature, it is shown that one of the two parameters appearing in the model can be considered independent of fibre and matrix materials for fibre dominated laminates, and is not affected by the shape of discontinuity. The proposed model can be therefore reduced to a one-parameter model.

A comparison with the WN criteria shows that the difference in the prediction of residual strength can be appreciated for large holes.

2. Theoretical considerations

Linear elastic fracture mechanics provide the following formula to predict the critical stress σ_c of a wide sheet containing a notch of length $2L$:

$$\sigma_c = K_{Ic}(\pi L)^{-1/2} \quad (1)$$

where K_{Ic} is the fracture toughness of the material.

It is important to note that Equation 1 is obtained under the hypothesis that the material is linearly elastic up to fracture. In fact, in composite materials a pseudo-plastic zone is formed at the crack tip during loading, giving rise to levelling of stresses ahead of the crack.

The shifting of material behaviour from linearity affects the exponent in Equation 1; the absolute

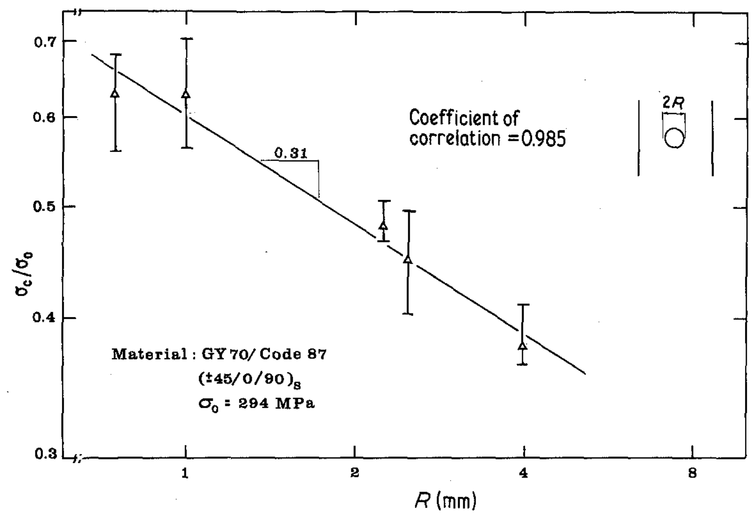

Figure 1 Log–log plot of the ratio σ_c/σ_0 between notched and unnotched strength against hole radius R for the GY 70/Code 87 $(\pm 45/0/90)_s$ laminate.

value of the exponent is expected to be lower for the pseudo-plastic case.

In general Equation 1 can be rewritten as:

$$\sigma_c = K_{Ic}(\pi L)^{-m} \tag{2}$$

which is identical to the formula proposed by Mar and Lin [7].

We shall now suppose that the fracture of an unnotched material is precipitated by an intrinsic flaw of length L_0; in this case from Equation 2:

$$\sigma_0 = K_{Ic}(\pi L_0)^{-m} \tag{3}$$

where σ_0 is the strength of the material. Finally from Equations 2 and 3

$$\sigma_c/\sigma_0 = (L_0/L)^m \tag{4}$$

Equation 4 provides a two-parameter model to predict the residual strength of a notched laminate.

3. Materials and test methods

Starting from GY 70/Code 87 epoxy resin by Fothergill and Harvey, six square panels 200 mm × 200 mm were press cured, following the procedure recommended by the supplier; a $(\pm 45/0/90)_s$ lamination sequence was chosen. Nominal thickness of the laminate was 0.6 mm.

Note that GY 70 fibres have a tensile modulus of ~607 × 10^3 MPa, about three times the one of T300 fibres.

Tensile specimens 25 mm wide, according to ASTM D 3039, were cut from each panel. Five samples were used to measure the unnotched strength, σ_0. Holes of radius R between 0.75 and 4 mm were introduced in some tensile specimens to ascertain the effect of holes on the laminate notched strength, σ_c; three valid tests were performed for each diameter. All tests were carried out on an Instron 1251 testing machine in stroke control, with a crosshead spead of 0.1 mm min^{-1}. A calibration factor [8] was used to account for the finite size of the specimens with holes.

4. Results and discussion

In order to verify the validity of Equation 4, proof test results were plotted in a log–log form, as shown in Fig. 1. It follows from Equation 4:

$$\log(\sigma_c/\sigma_0) = m \log L_0 - m \log L \tag{5}$$

Therefore the effect of the hole radius R on the fracture stress would be indicated, following Equation 5, by a straight line with a negative slope m.

The straight line drawn in Fig. 1 was obtained by a linear regression method. The coefficient of correlation is 0.985; the slope is negative and equal to 0.31. Therefore the correlation between experimental data and Equation 4 seems to be quite good, and m is lower than 0.5 as expected.

In [7] the results of tests carried out on 25 mm wide B/Al $(\pm 45/0)_s$ centre hole specimens are reported. The authors noted that a log–log plot of the quantity σ_c/σ_0 against hole radius R was well fitted (coefficient of correlation was 0.97) by a straight line having negative slope m equal to 0.30.

Despite the difference between material systems and laminates used in [7] and in the present work, the value of m seems to be quite constant. In order to verify the reliability of this observation, the data reported in [6] were used. They

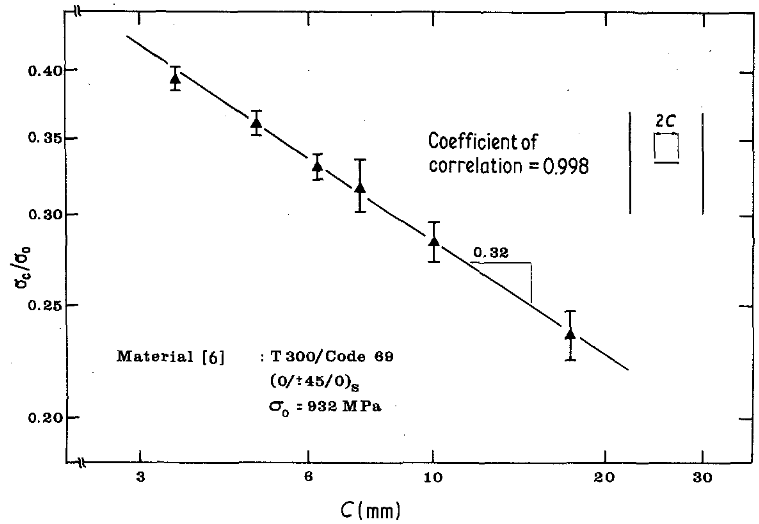

Figure 2 Log–log plot of the ratio σ_c/σ_0 between notched and unnotched strength against notch half-length C for the T300/Code 69 $(0/\pm45/0)_s$ laminate.

refer to two kinds of laminates, $(0/\pm45/0)_s$ and $(0/90)_{2s}$, respectively made with T300/Code 69 and T300/914 C prepreg. The tests were performed on centre-notched specimens of constant ratio $2C/W$ between notch length $2C$ and width W, and varying notch length in the range 5 to 35 mm. The authors noted that the WN criteria would not be able to describe the notch length effect in this case because of the increase of characteristic dimensions at increasing notch length.

In Figs. 2 and 3 the data reported in [6] are shown on a log–log scale respectively for $(0/\pm45/0)_s$ and $(0/90)_s$ laminates. The best fit straight lines have a negative slope of 0.32 and 0.31, respectively, with a very high coefficient of correlation (0.998 and 0.988). The values of m measured for all the laminates taken into consideration are reported in Table I.

The variation of m in a very narrow range of values (0.30 to 0.32) can of course be attributed to the scatter in experimental data; in this case Equation 4 is reduced to a one-parameter model, assuming $m = 0.31$.

It can be concluded that m can be considered as a constant for fibre dominated laminates, irrespective of material system, type of laminate and shape of discontinuity.

Following Equation 4 the length L_0 of the intrinsic flaw was calculated for all the laminates presented in this work (results are summarized in Table I). The results show a limited variation of L_0 for graphite/epoxy composites, despite the very different properties of T300 and GY 70 fibres. An appreciable increase in L_0 is measured for the B/Al laminate.

Note that the L_0 values are very low for all the

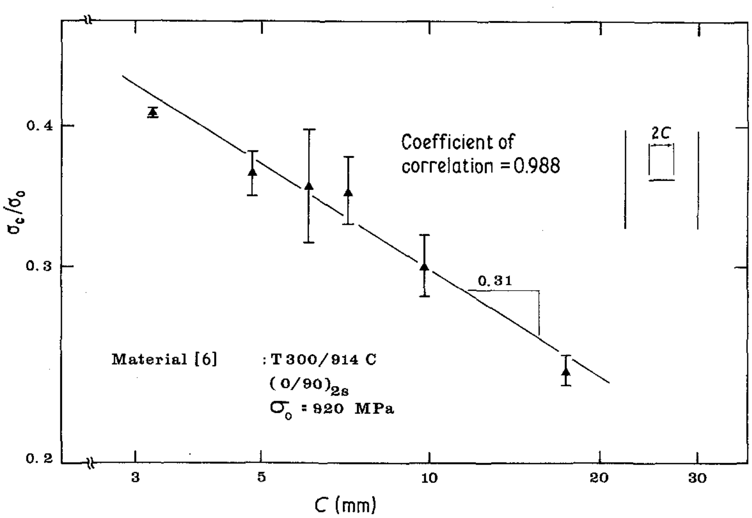

Figure 3 Log–log plot of the ratio σ_c/σ_0 between notched and unnotched strength against notch half-length C for the T300/914 C $(0/90)_s$ laminate.

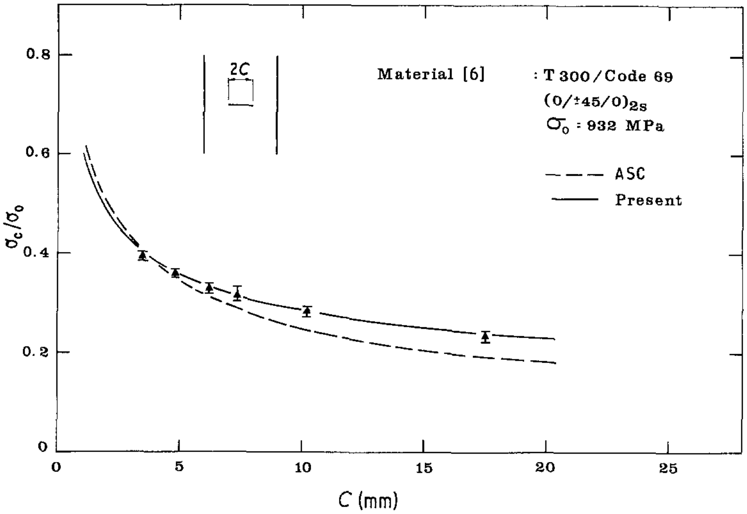

Figure 4 Strength reduction against notch-half-length C for the T300/Code 69 $(0/\pm 45/0)_s$ laminate.

materials considered; this supports the hypothesis that L_0 has the physical meaning of an intrinsic defect in the material.

Fig. 4 shows the measured values of σ_c/σ_0 plotted against $2C$ for the T300/Code 69 $(0/\pm 45/0)_s$ composite [6]. The broken line was drawn using the average stress criterion (ASC), where the value $d_0 = 0.42$ mm, as measured in [6] for this laminate, was adopted. The full curve represents Equation 4, where m and L_0 values of Table I were used.

It can be noted that an appreciable difference between the ASC and Equation 4 arises only for relatively large notches ($C \gtrsim 7$ mm), whereas for small notches the two criteria provide very close predictions. Similar results can be obtained using point stress or WEK criteria.

Equation 4 is therefore recommended when the residual strength of laminates containing large notches or holes must be predicted starting from experimental results obtained on small notches or holes. WN criteria however appear adequate for the purposes of analysis when small holes are under consideration.

5. Conclusions

A two-parameter model, similar to the one proposed by Mar and Lin [7], has been presented for prediction of residual strength of notched laminates. It has been shown by experimental data that one of the parameters appearing in the model seems to be independent of the material system, type of laminate and shape of discontinuity for fibre dominated materials; the other parameter does depend on the material under examination, and can probably be related to real defects present in the material itself.

A comparison between experimental data and predictions based on the WN criteria and the present model shows that WN criteria are reliable for predicting residual strength when small notches or holes are considered. However, when large notches or holes are dealt with, the model presented in this work seems to be more efficient.

TABLE I Measured values of m and L_0 for all laminates presented in this work

Reference	Material	Laminate	m	L_0 (mm)
Present	GY 70/87	$(\pm 45/0/90)_s$	0.31	0.196
[6]	T300/69	$(0/\pm 45/0)_s$	0.32	0.200
[6]	T300/914 C	$(0/90)_{2s}$	0.31	0.214
[7]	B/Al	$(\pm 45/0)_s$	0.30	0.314

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